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NONLINEAR HEAT TRANSFER AND TEMPERATURE DISTRIBUTION THROUGH FINS AND ELECTRIC FILAMENTS OF ARBITRARY GEOMETRY WITH TEMPERATURE-DEPENDENT PROPERTIES AND HEAT GENERATION

By A. R. Shouman
Propulsion and Vehicle Engineering Laboratory

NASA

George C. Marshall Space Flight Center, Huntsville, Alabama

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ABSTRACT

An analytical treatment is presented for the nonlinear differential equation governing the one-dimensional steady-state temperature distribution along a rod or a fin due to the heat exchange between them and the surroundings by both convection and radiation. The analysis treats the problem in general and includs variation in geometry, dependence of properties on temperature, and internal heat generation.

It was shown that the special case of constant area geometry could be solved exactly including temperature dependent physical properties and with heat generation that at most is temperature dependent only. The solution, however, has to be determined for each case separately depending on the functions describing the dependence of the physical properties and the heat generation on temperature.

The solution for the constant area geometry with constant thermal properties leads to the definition of two functions in terms of integrals. The integrals can be evaluated numerically to any desired accuracy and the functions will be tabulated and published under separate cover.

For the general case of arbitrary geometry and with physical properties and heat generation that is location dependent as well as temperature dependent, a new method for solving the problem numerically is outlined.

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A. R Shouman

APPLIED RESEARCH BRANCH
PROPULSION DIVISION
PROPULSION AND VEHICLE ENGINEERING LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

TABLE OF CONTENTS

DEFINITION OF SYMBOLS

Symbol	Definition
A	Area, ft^2
C	Constant
C'	Constant
D	Diameter, ft
E	Emissivity factor, dimensionless
f	Function
g	Constant
h	Convection heat transfer coefficient, Btu/hr ft^2 °R
k	Thermal conductivity, Btu/hr ft °R
L	Length, ft
P	Perimeter, ft
Q	Rate of heat transfer, Btu/hr
q'''	Internal heat generation, Btu/hr ft^3
r	Constant, $\frac{5h}{2\sigma E T_o{}^3}$
S	Length along surface, ft
T	Temperature, °R
V	Volume, ft^3
x	Distance, ft
α	Constant
δ	Half thickness of rectangular fin, ft
θ	Dimensionless variable, T/T_o
λ	$1/\theta$
Λ	Function
Φ	Function

DEFINITION OF SYMBOLS (Concluded)

Subscripts

Symbol	Definition
L	Condition at a distance L from origin
o	Condition at origin
i	Condition at origin for infinite fin
s	Condition of surroundings
sr	Condition of surroundings due to radiation
sc	Condition of surroundings due to convection

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George C. Marshall Space Flight Center
Huntsville, Alabama

SUMMARY

A theoretical analysis was conducted of one-dimensional, steady-state, heat exchange by both convection and radiation between a rod or a fin and their surroundings. The analysis considers both the cases where the heat is being dissipated or received by the fin or rod. Both the infinite and finite length cases are considered. The surroundings equivalent temperature for radiation could be different from that for convection. The fin or rod could have an arbitrary geometry and its physical properties could be temperature dependent as well as displacement dependent. The analysis also considers the effect of heat generation or absorption.

The exact solution of the problem for a constant area fin or rod with constant thermal properties is presented. The solution produces two functions of parametric nature. The different parameters are dependent on the boundary conditions. A computer program for evaluating the functions was written by James W. Price of the Applied Research Branch. The program (Appendix A) can be used directly for solving fin problems. However, to make this study complete and of use for those to whom a computer is not readily available, the two functions will be

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* Professor of Mechanical Engineering, New Mexico State University, Las Cruces, New Mexico. Senior Postdoctoral Resident Research Associate of the National Research Council, National Academy of Sciences and National Academy of Engineering at the NASA/George C. Marshall Space Flight Center, Huntsville, Alabama in the Propulsion and Vehicle Engineering Laboratory.

presented in tabulated and graphic forms with examples in a later report.

It was shown that the heat generation or absorption cases are reducible to the simple case through a suitable transformation. After this transformation, the tabulated functions can be used for solving problems with internal heat generation absorption.

The constant area problem with temperature-dependent properties is also solved exactly. The solution, however, is dependent on the functions describing the dependence of the physical properties of temperature. The solution is presented in the form of integrals that can be evaluated for any case under consideration.

The problem in general was examined, and the conditions under which it could be solved exactly were outlined together with the method of solution. A procedure for numerically solving the problem when exact solutions are not possible was outlined. The procedure differs from the point-slope method generally used in the literature for handling this problem by choosing the origin where the temperature gradient is specified instead of the customary choice of the origin at the point where the temperature is specified.

INTRODUCTION

The subject of heat transfer from fins and extended surfaces has been studied analytically and experimentally for almost two centuries [1]. Most work has been concentrated on convection fins with constant thermal properties. The work on convection fins was culminated and summarized by Gardner [2]. The problem of a fin dissipating heat to the surroundings by radiation has recently come under extensive study because of the interest in space and space travel.

Numerical solutions of the radiating fin problem using difference equations and computers are given in the literature by Chambers and Somers [3], Lieblein [4], Bartas and Sellers [5] and Callinan and Berggren [6]. Wilkins [7] and Liu [8] treated the problem of the minimum mass fin. Mackay [9] outlined a method of successive approximations to be used in connection with a digital computer. Shouman [10] presented an exact solution for the problem. Stockman and Kramer [11] considered the effects of linearly varying conductivity and emissivity for the heat transfer of radiation fins using the Runge-Kutta method. Since convection heat transfer sometimes accounts for a significant portion of the heat exchange, it is necessary to examine the problem when both convection and radiation are present. Cobble [12] examined the constant area fin with temperature dependent properties and combined convection and radiation. His solution was obtained in terms of Jacobian elliptic functions with the aid of the Gregory-Newton Forward Interpolation formula. Shouman [13] presented an exact solution for the constant area case with constant thermal properties and with combined convection and radiation heat transfer. Hung and Appl [14] presented a mathematical iteration technique for handling the general fin problem and considered the effect of variation of thermal properties with temperature and the question of heat generation. Shouman [15] presented an exact solution for the constant area fin with temperature dependent properties and temperature dependent heat generation with combined convection and radiation heat transfer.

The distribution of temperature along a thin rod or a thin-walled tube heated by passing a heavy electric current through it has also been of interest, both experimentally and theoretically, for a long time. The work on this problem has been concentrated on the case where the rod or tube was maintained in a vacuum, namely, when the heat dissipated from the surface by radiation only. The literature on this problem contains work by Langmuir [16], Langmuir and Taylor [17], Worthing [18-20], Worthing and Holliday [21], Stead [22], Bush and Gould [23], Prescott and Hincke [24], Baerwald [25], Jain and Krishnan [26-31], and others. The differential equation describing this problem is identical to those describing the problem of a fin receiving heat by radiation only from surroundings whose exact solution is presented by Shouman [32].

The purpose of this study was to examine the differential equation describing the one-dimensional temperature distribution and heat transfer along fins or electrical filaments of variable cross-sectional area and geometry, with temperature dependent properties and internal heat generation or absorption. The heat exchange with the surroundings can be by both radiation and convection. The heat exchange can be from the fin to the surroundings or vice versa. Both the infinite and finite geometries were considered. The conditions required for exactly solving the differential equation were also examined. When the exact solution was not possible, an analytical procedure was used that greatly simplified the solution of the problem using numerical methods.

In this report the constant area fin or filament problem with constant thermal properties is dealt with in detail. It is shown that the problem can be solved exactly. The solution produces two functions with four parameters that can be tabulated. These two functions are presented in tabulated and graphical forms in another report together with examples on how to use the tables and graphs to solve different problems.

Acknowledgement

The author is indebted to his colleague Dr. M. H. Cobble for bringing this problem to his attention during an oral examination and for his valuable discussions. The support of the Engineering Experiment Station of New Mexico State University during the early stage of this work is acknowledged. It would have been impossible to complete the early computer programming without the support of Mr. P. C. Fang, Graduate Assistant, New Mexico State University. This work was brought to completion while the author pursued a National Academy of Science, National Research Council and National Academy of Engineering Postdoctoral Resident Research Associateship at the Propulsion and Vehicle Engineering Laboratory of NASA/George C. Marshall Space Flight Center. The support given by the NASA personnel is greatly appreciated.

MATHEMATICAL ANALYSIS

The differential equation describing the one-dimensional steady-state temperature distribution and heat flow under the three modes of heat transfer and with heat generation is well established. It may be written in the following form

$$\frac{d}{dx}\left[A(x)\,k(T)\,\frac{dT}{dx}\right] + q'''(x, T)\,A(x) - P(x)\,\frac{dS}{dx}\left[\sigma E(T, x)\,(T^4 - T_s^{\,4}) + h(T, x)\,(T - T_s)\right] = 0 \tag{1}$$

where:

x	=	distance along x axis.
$A(x)$	=	area of fin perpendicular to heat flow.
$k(T)$	=	thermal conductivity of fin material, which is temperature dependent.
$T(x)$	=	local temperature of fin at x.
$q\ (x, T)$	=	rate of internal heat generation per unit volume at x, which is dependent on both x and T.
$P(x)$	=	perimeter of the fin at x.
dS	=	element of length along the surface of the fin.
σ	=	Stefan - Boltzmann constant.
$E(T, x)$	=	emmissivity of the fin surface at x, which is also dependent on T.
T_s	=	equivalent surrounding temperature.
$h(T, x)$	=	convection film coefficient at x, which is also temperature dependent.

Before we proceed with our analysis, a few comments about equation (1) are in order. The difference between $\frac{dS}{dx}$ and unity is customarily ignored in the analysis of thin fins. However, it will be considered in this analysis. The equivalent surrounding temperature in the radiation term can be determined taking into account all incident radiation from the surroundings. In general, the equivalent surrounding temperature for the radiation term can be different than that for the convection term. It will be shown that, through a simple transform, the equation can be reduced to the form where the equivalent surrounding temperatures for both radiation and convection are the same. The equivalent surrounding temperatures for radiation and convection will be considered to be identical and later the necessary transformation will be introduced. The equivalent surrounding temperature will be treated as an independent variable to facilitate the mathematical analysis. In reality, however, the equivalent surrounding temperature due to the radiation field depends on many factors. The details of determining the equivalent surrounding temperature for some configurations are considered, for example, in Reference 4.

Although the proportionality of the heat exchange by radiation to the fourth power of the temperature has been established on theoretical and experimental bases for black body radiation, there have been questions raised about its validity for metallic surfaces. Using electromagnetic theory, Aschkinass [31] concluded that for electrical conductors, the exponent over the temperature should be 4.5. Jacob [1], with slight modification of the constants used in the derivation, suggests an exponent of 5.0. Langmuir [16] reports an exponent of 4.96 for tungsten between 400°K and 2400°K based on experimental evidence. Although the exponent used in this analysis is 4, any other exponent can readily be used. It is hoped that with the solution of the equation for the temperature distribution, it will be easy to determine the exponent experimentally and examine the validity of the different theories.

To make clear the method of analysis for equation (1) that will be recommended later, the special case of the constant area fin will be considered first.

THE CONSTANT AREA FIN CASE

Constant Thermal Properties Without Heat Generation

If a constant area fin is considered without any heat generation and with the thermal properties constant, equation (1) becomes:

$$\frac{d^2T}{dx^2} - \frac{\sigma EP}{kA}(T^4 - T_s^4) - \frac{hP}{kA}(T - T_s) = 0 \qquad (2)$$

Equation (2) can readily be integrated once giving:

$$\left(\frac{dT}{dx}\right)^2 = \frac{2\sigma EP}{5kA}(T^5 - 5T_s^4 T) + \frac{hP}{kA}(T^2 - 2T_s T) + C' \qquad (3)$$

The solution of equation (3) is normally required, subject to the boundary conditions $T = T_1$ at $x = 0$ and $\frac{dT}{dx} = 0$ at $x = L$, because of symmetry or insulation. In general, however, the boundary condition at $x = L$ can be given by $\left(\frac{dT}{dx}\right)_{x = L} = f(T_L)$ where other considerations like the rate of heat exchange at $x = L$ must be applied to determine the function $f(T_L)$. Before considering equation (3) the special case of the infinite fin with zero equivalent surrounding temperature will be considered, since the solution can be readily obtained through integration.

The Infinite Fin With Zero Equivalent Sink Temperature

For this case $T_s = 0$, which reduces equation (3) to:

$$\left(\frac{dT}{dx}\right)^2 = \frac{2\sigma EP}{5kA} T^5 + \frac{hP}{kA} T^2 + C' \tag{4}$$

Equation (4) is subject to the boundary conditions.

$T = T_1$ at $x = 0$

and

$\frac{dT}{dx} = 0$ at $x = \infty$

The second boundary condition requires that

$T = 0$ at $x = \infty$

Applying this condition to equation (4) yields $C' = 0$, giving:

$$\frac{dT}{dx} = -T\left(\frac{2\sigma EP}{5kA} T^3 + \frac{hP}{kA}\right)^{\frac{1}{2}} \tag{5}$$

The negative sign is used because T decreases with increasing x. Equation (5) can be directly integrated to give the final result:

$$x = \frac{1}{3}\left(\frac{kA}{hP}\right)^{\frac{1}{2}}\left[\ln \frac{\left(\frac{2\sigma EP}{5kA} T_1{}^3 + \frac{hP}{kA}\right)^{\frac{1}{2}} - \left(\frac{hP}{kA}\right)^{\frac{1}{2}}}{\left(\frac{2\sigma EP}{5kA} T_1{}^3 + \frac{hP}{kA}\right)^{\frac{1}{2}} + \left(\frac{hP}{kA}\right)^{\frac{1}{2}}} - \ln \frac{\left(\frac{2\sigma EP}{5kA} T^3 + \frac{hP}{kA}\right)^{\frac{1}{2}} - \left(\frac{hP}{kA}\right)^{\frac{1}{2}}}{\left(\frac{2\sigma EP}{5kA} T^3 + \frac{hP}{kA}\right)^{\frac{1}{2}} + \left(\frac{hP}{kA}\right)^{\frac{1}{2}}}\right] \tag{6}$$

which can also be written as:

$$x = \left(\frac{kA}{hP}\right)^{\frac{1}{2}} \left[\ln \frac{T_1}{T} - \frac{2}{3} \ln \frac{\left(\frac{2\sigma EP}{5kA} T_1^{\,3} + \frac{hP}{kA}\right)^{\frac{1}{2}} + \left(\frac{hP}{kA}\right)^{\frac{1}{2}}}{\left(\frac{2\sigma EP}{5kA} T^3 + \frac{hP}{kA}\right)^{\frac{1}{2}} + \left(\frac{hP}{kA}\right)^{\frac{1}{2}}} \right] \tag{7}$$

In the absence of convection, both equation (6) and (7) are not very useful. Equation (5) reduces to

$$\frac{dT}{dx} = - \left(\frac{2\sigma EP}{5kA}\right)^{\frac{1}{2}} T^{\frac{5}{2}} \tag{8}$$

which can be integrated directly to give:

$$\left(\frac{2\sigma EP}{5kA}\right)^{\frac{1}{2}} x = \frac{2}{3} \left(\frac{1}{T^{\frac{3}{2}}} - \frac{1}{T_1^{\frac{3}{2}}}\right) \tag{9}$$

Equation (9) can also be written as

$$\frac{T}{T_1} = \left[\frac{1}{1 + \frac{3}{2} \left(\frac{2\sigma EP\ T_1^{\,3}}{5kA}\right)^{\frac{1}{2}} x} \right]^{\frac{2}{3}} \tag{10}$$

The Finite Equivalent Surrounding Temperature

When the equivalent surrounding temperature is finite, the zero x axis shall be chosen where the temperature gradient is specified instead of where the temperature is specified. The reason for this choice will be made clear as the analysis progresses. The positive x axis will then be in the direction of increasing temperature when the fin transfers heat to the surroundings and in the direction of decreasing temperature when the fin receives heat from the surrounding. Assuming the temperature at x = 0 to be T_O and substituting in equation (3), the following is obtained:

$$\left(\frac{dT}{dx}\right)^2_{x=0} = \frac{2\sigma EP}{5kA} (T_O^{\,5} - 5T_s^{\,4}\ T_O) + \frac{hP}{kA} (T_O^{\,2} - 2T_s T_O) + C' \tag{11}$$

$$\therefore \; C' = \left(\frac{dT}{dx}\right)^2_{x=0} - \frac{2\sigma EP}{5kA}\left(T_o^5 - 5T_s^4\, T_o\right) - \frac{hP}{kA}\left(T_o^2 - 2T_s\, T_o\right) \tag{12}$$

$$\therefore \; \left(\frac{dT}{dx}\right)^2 - \left(\frac{dT}{dx}\right)^2_{x=0} = \frac{2\sigma EP}{5kA}\left[T^5 - T_o^5 - 5T_s^4\,(T - T_o)\right] \tag{13}$$

$$+ \frac{hP}{kA}\left[T^2 - T_o^2 - 2T_s\,(T - T_o)\right]$$

Introducing the variable $\theta = T/T_O$ and $r = 5h/2\sigma E T_O{}^3$ gives

$$\left(\frac{d\theta}{dx}\right)^2 - \left(\frac{d\theta}{dx}\right)^2_{x=0} = \frac{2\sigma EP\, T_O{}^3}{5kA}\left[(\theta^5 - 1) - 5\theta_s^4\,(\theta - 1)\right. \tag{14}$$

$$\left. + r(\theta - 1)\,(\theta - 2\theta_s + 1)\right]$$

assuming $\left(\frac{d\theta}{dx}\right)^2_{x=0} = g\left(\frac{2\sigma EP\, T_O^3}{5kA}\right)$, then

$$\frac{d\theta}{dx} = \pm\left(\frac{2\,\sigma EP\, T_O^3}{5kA}\right)^{\frac{1}{2}}\left[(\theta^5 - 1) - 5\theta_s^4\,(\theta - 1)\right. \tag{15}$$

$$\left. + r(\theta - 1)(\theta - 2\theta_s + 1) + g\right]^{\frac{1}{2}}$$

The solution to equation (15) can be written as:

$$\left(\frac{2\sigma EP\, T_O{}^3}{5kA}\right)^{\frac{1}{2}} x = \pm \int_1^{\theta}\left[(\theta^5 - 1) - 5\theta_s^4\,(\theta - 1)\right. \tag{16}$$

$$\left. + r(\theta - 1)(\theta - 2\theta_s + 1) + g\right]^{-\frac{1}{2}} d\theta$$

Substituting for $C = g + 5\theta_s^4 - 1 + r(2\theta_s - 1)$, equation (16) can be written as follows:

$$\left(\frac{2\sigma EP\, T_o^3}{5kA}\right)^{\frac{1}{2}} x = \pm \int_1^{\theta} \left[\theta^5 - 5\theta_s^4\, \theta + r(\theta^2 - 2\theta_s\theta) + C\right]^{-\frac{1}{2}} d\theta \tag{17}$$

The solution to equation (17) is subject to the boundary condition

$$\left(\frac{2\sigma EP\, T_o^3}{5kA}\right)^{\frac{1}{2}} L = \pm \int_1^{\theta_L} \left[\theta^5 - 5\theta_s^4 \theta + r(\theta^2 - 2\theta_s\theta) + C\right]^{-\frac{1}{2}} d\theta \tag{18}$$

where L is the length at which $\theta = \theta_L = T_L/T_o$. Normally T_L is known, not T_o. Once the value of T_o required to satisfy equation (18) is obtained, the temperature at any point in the rod can be obtained from equation (17).

The positive and negative signs in equations (15), (16), (17) and (18) correspond to the cases of heat transfer to and from the surroundings. In general, the positive sign corresponds to the case where θ increases in the increasing direction of x while the negative sign corresponds to the case where θ decreases in the increasing direction of x. The integral on the right hand side of equation (17) could not be evaluated in terms of the simple functions. It could be evaluated numerically or graphically very readily for $g > 0$. For $g = 0$, it can be seen from equation (16) that a singularity exists at $\theta = 1$, which calls for special consideration for evaluation of the integral. We shall define the two functions Φ and Λ as follows:

$$\Phi\,(\theta,\ \theta_s,\ r,\ C) = \int_{\theta}^{\infty} \left[\theta^5 - 5\theta_s^4\, \theta + r\,(\theta^2 - 2\theta_s\theta) + C\right]^{-\frac{1}{2}} d\theta \tag{19}$$

for $\quad 0 \leqq \theta_s \leqq 1.0, \quad \theta \geqq 1.0$, and

$$\Lambda(\theta,\ \theta_s,\ r,\ C) = \int_0^{\theta} \left[\theta^5 - 5\theta_s^4\theta + r(\theta^2 - 2\theta_s\theta) + C\right]^{-\frac{1}{2}} d\theta \tag{20}$$

for $\quad \theta_s \geqq 1.0 \quad$ and $\quad \theta \leqq 1.0$

If both Φ and Λ are proved to be finite in the range of interest, the solution can be written for the fin transferring heat as

$$\left(\frac{2\sigma EP\, T_o^{\,3}}{5kA}\right)^{\frac{1}{2}} x = \Phi(1, \theta_s, r, C) - \Phi(\theta, \theta_s, r, C) \tag{21}$$

and for the fin receiving heat as

$$\left(\frac{2\sigma EP\, T_o^{\,3}}{5kA}\right)^{\frac{1}{2}} x = \Lambda(1, \theta_s, r, C) - \Lambda(\theta, \theta_s, r, C) \tag{22}$$

Evaluation of The Functions Φ and Λ

An examination of Φ and Λ shows that both functions have an upper bound which exists for $r = 0$, $g = 0$ and $\theta_s = 1$ and also that the functions are finite in the entire field except at the point $\theta = 1$ when $\theta_s = 1$ and $g = 0$. The case of $\theta_s = 1$ and $g = 0$ represents an infinite fin with an insulated end and it shall be considered separately.

The Function Φ

Introducing the variable $\lambda = 1/\theta$, equation (19) can be written as follows:

$$\Phi(\lambda, \theta_s, r, C) = 2/3 \int_0^{\lambda} \left[1 - 5\theta_s^{\,4}\lambda^4 + r\lambda^3(1 - 2\theta_s\lambda) + C\lambda^5\right]^{-\frac{1}{2}} d(\lambda^{\frac{3}{2}}) \tag{23}$$

Equation (23) can be used to evaluate Φ when $g > 0$. The integral can be evaluated to any desired accuracy using a suitable numerical scheme and utilizing $\lambda^{3/2}$ as the independent variable. To remove the singularity at $\lambda = 1$ when $g = 0$, the integral is first integrated by parts leading to the following:

$$\Phi(\lambda, \theta_s, r, C) = 8/3 \int_0^{\lambda} \frac{[1 - 5\theta_s^{\,4}\lambda^4 + r\lambda^3(1 - 2\theta_s\lambda) + C\lambda^5]^{\frac{1}{2}}(10 + r\lambda^3)\, d(\lambda^{\frac{3}{2}})}{[5(1 - \theta_s^{\,4}\lambda^4) + 2r\lambda^3(1 - \theta_s\lambda)]^2}$$
$$- \frac{2\lambda^{\frac{3}{2}}[1 - 5\theta_s^{\,4}\lambda^4 + r\lambda^3(1 - 2\theta_s\lambda) + C\lambda^5]^{\frac{1}{2}}}{5(1 - \theta_s^{\,4}\lambda^4) + 2r\lambda^3(1 - \theta_s\lambda)} \tag{24}$$

It can be seen from equation (24) that Φ is finite in the whole range of interest except at $\lambda = 1$, $\theta_s = 1$, and $g = 0$, which will be discussed later. A numerical scheme similar to that used with equation (23) can be used for evaluating Φ using equation (24).

The Function Λ

When $g > 0$, Λ can be calculated using equation (20). When $g = 0$, integration by parts removes the singularity at $\theta = 1$ and produces the following:

$$\Lambda(\theta, \theta_s, r, C) = \frac{2[\theta^5 - 5\theta_s^4\theta + r(\theta^2 - 2\theta_s\theta) + C]^{\frac{1}{2}}}{5(\theta^4 - \theta_s^4) + 2r(\theta - \theta_s)} + \frac{2C^{\frac{1}{2}}}{5\theta_s^4 + 2r\theta_s} \tag{25}$$

$$+ 4\int_0^\theta \frac{[\theta^5 - 5\theta_s^4\theta + r(\theta^2 - 2\theta_s\theta) + C]^{\frac{1}{2}}(10\theta^3 + r)\, d\theta}{[5(\theta^4 - \theta_s^4) + 2r(\theta - \theta_s)]^2}$$

It can be seen from (25) that Λ is finite in the entire range of interest except at $\theta = 1$ for the infinite insulated end fin where $\theta_s = 1$ and $g = 0$.

Both the functions Φ and Λ were evaluated using a digital computer for different values of the parameters θ_s, r, and g. They will be presented in tabular and graphic forms in a later report.

The preceding analysis applies for all positive values of r. The limiting value of $r = \infty$, reduces the problem to the well known linear convection problem.

The Negative Values of r - It is possible for r to assume negative as well as positive values. However, it will appear from the following consideration that r has a minimum value. Equation (2) for negative r can be written as:

$$\frac{d^2T}{dx^2} = \frac{\sigma EP}{kA}(T^4 - T_s^4) - \frac{hP}{kA}(T - T_s) \tag{26}$$

There is a temperature T_m defined by

$$\frac{\sigma EP}{kA}(T_m^4 - T_s^4) = \frac{hP}{kA}(T_m - T_s) \tag{27}$$

or

$$(T_m + T_s)(T_m^2 + T_s^2) = \frac{h}{\sigma E} \tag{28}$$

T_m is the minimum or maximum temperature that can be reached in an infinite fin at the point where the temperature gradient is zero. Using T_m for T_o gives

$$(1 + \theta_s)(1 + \theta_s^2) = \frac{h}{\sigma E T_o^3} = \frac{2}{5} r \tag{29}$$

Therefore,

$$r_{min} = - \frac{5}{2} (1 + \theta_s)(1 + \theta_s^2) \tag{30}$$

However, it can be shown that Λ is imaginary for

$$r < \frac{1 - 5\theta_s^4}{2\theta_s - 1} \quad \text{when } g = 0$$

The Case of $r = -1$ - The case of $r = -1$ will be considered separately, since it can be integrated in terms of simple functions under certain conditions. For $r = -1$ and $(d\theta/dx)_{x=0} = 0$, equation (14) reduces to

$$\left(\frac{d\theta}{dx}\right)^2 = \left(\frac{2\sigma EP\, T_o^3}{5kA}\right) [\theta^5 - \theta^2 - (5\theta_s^4 - 2\theta_s)\theta + 5\theta_s^4 - 2\theta_s] \tag{31}$$

For $\theta_s = 0$ or $\theta_s^3 = 0.4$, equation (31) becomes

$$\left(\frac{d\theta}{dx}\right)^2 = \left(\frac{2\sigma EP\, T_o^3}{5kA}\right) \theta^2 (\theta^3 - 1) \tag{32}$$

Equation (32) can be integrated directly giving

$$\pm \left(\frac{2\sigma EP\, T_o^3}{5kA}\right)^{\frac{1}{2}} x = \frac{2}{3} \tan^{-1} (\theta^3 - 1)^{\frac{1}{2}} \tag{33}$$

which can be written as

$$\pm \left(\frac{9}{10} \frac{\sigma EP\, T_o^3}{kA}\right)^{\frac{1}{2}} x = \sec^{-1} \theta^{\frac{3}{2}} = \cos^{-1} \lambda^{\frac{3}{2}} \tag{34}$$

The Infinite Fin Case - For the sake of completeness, the infinite fin case and g = 0 will be considered. For r positive, this represents the case where $\theta_s = 1$ or $T_o = T_s$, and the solution becomes

$$\left(\frac{2\sigma EP\, T_s^3}{5kA}\right)^{\frac{1}{2}} \Delta x = \Phi\,(\theta_2,\ 1,\ r,\ g = 0) - \Phi\,(\theta_1,\ 1,\ r,\ g = 0) \tag{35}$$

for the fin dissipating heat and

$$\left(\frac{2\sigma EP\, T_s^3}{5kA}\right)^{\frac{1}{2}} \Delta x = \Lambda(\theta_2,\ 1,\ r,\ g = 0) - \Lambda(\theta_1,\ 1,\ r,\ g = 0) \tag{36}$$

for the fin receiving heat.

For any initial condition T_1 and T_s, T_2 can be evaluated for any change in length Δx.

For negative r, the infinite fin case is where $r = r_{min} = -\ 5/2\ (1 + \theta_s)(1 + \theta_s^2)$ and $T_o = T_m$. T_m can be calculated from equation (28) and the solution becomes

$$\left(\frac{2\sigma EP\, Tm^3}{5kA}\right)^{\frac{1}{2}} \Delta x = \Phi\,(\theta_2,\ \theta_s,\ r_{min},\ g = 0) - \Phi\,(\theta_1,\ \theta_s,\ r_{min},\ g = 0) \tag{37}$$

for the fin dissipating heat and

$$\left(\frac{2\sigma EP\, Tm^3}{5kA}\right)^{\frac{1}{2}} \Delta x = \Lambda(\theta_2,\ \theta_s,\ r_{min},\ g = 0) - \Lambda(\theta_1,\ \theta_s,\ r_{min},\ g = 0) \tag{38}$$

for the fin receiving heat.

For any initial condition T_1 and T_s, T_2 can be evaluated for any change in length Δx.

Heat Transfer Calculations - With the solution of the equations, the calculations of the heat transfer, the fin effectiveness and other quantities of interest follow readily. To illustrate this, the problem of the minimum mass fin will be considered.

THE MINIMUM MASS FIN

As an example of the use of the solution, consider a constant area fin with constant base temperature, T_L, that is required to transfer an amount of heat, Q. It is quite often desirable to obtain the conditions required to minimize the mass of the fin. It can be readily shown that:

$$Q = - (2/5\ \sigma EkPA)^{\frac{1}{2}}\ T_L^{\frac{5}{2}}\ [1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3\ (1 - 2\theta_s\lambda_L) + C\lambda_L^5]^{\frac{1}{2}} \tag{39}$$

This gives

$$A = \frac{5Q^2}{2\sigma EkPT_L^5[1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3(1 - 2\theta_s\lambda_L) + C\lambda_L^5]} \tag{40}$$

and

$$L = \left(\frac{5kA}{2\sigma EPT_L^3}\right)^{\frac{1}{2}} \theta_L^{\frac{3}{2}}\ [\Phi(1,\ \theta_s,\ r,\ C) - \Phi(\lambda_L,\ \theta_s,\ r,\ C)] \tag{41}$$

For a rectangular fin of thickness 2δ, $P = 2$, and $A = 2\delta$, it follows:

$$\delta = \frac{5Q^2}{8\sigma EkT_L^5[1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3(1 - 2\theta_s\lambda_L) + C\lambda_L^5]} \tag{42}$$

and

$$L = \left(\frac{5k\delta}{2\sigma EPT_L^3}\right)^{\frac{1}{2}} \theta_L^{\frac{3}{2}}\ [\Phi(1,\ \theta_s,\ r,\ C) - \Phi(\lambda_L,\ \theta_s,\ r,\ C)] \tag{43}$$

By combining equations (42) and (43), the volume of the fin is obtained as

$$V = 2\delta L = \frac{25Q^3}{16\sigma^2E^2kT_L^9}\ \frac{[\Phi(1,\ \theta_s,\ r,\ C) - \Phi(\lambda_L,\ \theta_s,\ r,\ C)]}{\lambda_L^{\frac{3}{2}}[1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3(1 - 2\theta_s\lambda_L) + C\lambda_L^5]^{\frac{3}{2}}} \tag{44}$$

For V to be minimum, the expression

$$\frac{[\Phi(1,\ \theta_s,\ r,\ C) - \Phi(\lambda_L,\ \theta_s,\ r,\ C)]}{\lambda_L^{\frac{3}{2}}[1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3(1 - 2\theta_s\lambda_L) + C\lambda_L^5]^{\frac{3}{2}}}$$

should be a minimum. Differentiating and equating to zero gives the following equation:

$$\Phi(1,\ \theta_s,\ r,\ C) - \Phi(\lambda_L,\ \theta_s,\ r,\ C) \tag{45}$$

$$= -\frac{2}{3}\ \frac{\lambda_L^{\frac{3}{2}}[1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3(1 - 2\theta_s\lambda_L) + C\lambda_L^5]^{\frac{1}{2}}}{[1 - 25\theta_s^4\lambda_L^4 + 2r\lambda_L^3(2 - 5\theta_s\lambda_L) + 6C\lambda_L^5]}$$

The solution to equation (45) gives the conditions required for a minimum mass rectangular fin.

For a circular fin of diameter D, $A = \frac{\pi D^2}{4}$ and $P = \pi D$, the following expression is obtained.

$$V = \frac{1}{16}\left(\frac{10^4\ Q^5}{\pi^2\sigma^4 E^4 k T_L^{17}}\right)^{\frac{1}{3}} \frac{[\Phi(1,\ \theta_s,\ r,\ C) - \Phi(\lambda_L, \theta_s,\ r,\ C)]}{\lambda_L^{\frac{3}{2}}[1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3(1 - 2\theta_s\lambda_L) + C\lambda_L^5]^{\frac{5}{6}}} \tag{46}$$

Differentiating and equating to zero, the condition for minimum V is found to be

$$[\Phi(1,\ \theta_s,\ r,\ C) - \Phi(\lambda_L,\ \theta_s,\ r,\ C)] \tag{47}$$

$$= -\ \frac{\lambda_L^{\frac{3}{2}}[1 - 5\theta_s^4\lambda_L^4 + r\lambda_L^3(1 - 2\theta_s\lambda_L) + C\lambda_L^5]^{\frac{1}{2}}}{\frac{3}{2} - \frac{145}{6}\ \theta_s^4\lambda_L^4 + r\lambda_L^3(4 - \frac{29}{3}\ \theta_s\lambda_L) + \frac{17}{3}\ C\lambda_L^5}$$

The solution to equation (47) gives the requirements for minimum mass fin geometry.

Constant Thermal Properties with Heat Generation or Absorption

The problem of heat transfer from a constant area rod or tube with internal heat generation arises naturally in the case of the electric filament heated by the passage of an electric current. Similarly, the heat could be generated by an atomic or chemical reaction as in an atomic reactor or an exothermic chemical reaction. The case of heat absorption occurs when an endothermic chemical reaction or an evaporation process takes place within the tube. It will be shown that a simple transformation reduces this problem to the no-heat generation case considered earlier.

Constant Heat Generation

Assuming the heat generation term to be constant, equation (1) can be written as:

$$\frac{d^2T}{dx^2} - \frac{\sigma EP}{kA}(T^4 - T_s^4) - \frac{hP}{kA}(T - T_s) + \frac{q'''}{k} = 0 \qquad (48)$$

q''' is positive when the heat is generated and negative when the heat is absorbed. The temperature, T_m, can be defined by the following equation,

$$q''' = \frac{\sigma EP}{A}(T_m^4 - T_s^4) + \frac{hP}{A}(T_m - T_s) \qquad (49)$$

T_m is greater than T_s when q''' is positive and less than T_s when q''' is negative. Physically speaking T_m is the maximum or minimum temperature that can be reached in a fin or rod of infinite length. Substituting equation (49) into equation (48) results in equation (50)

$$\frac{d^2T}{dx^2} - \frac{\sigma EP}{kA}(T^4 - T_m^4) - \frac{hP}{kA}(T - T_m) = 0 \qquad (50)$$

Equation (50) is the same as equation (2) with T_m replacing T_s. Hence, the solution procedure is identical. The same procedure used above can be used to determine the equivalent surrounding temperature, T_s, of a space that has an equivalent radiation temperature, T_{sr}, and equivalent convection temperature, T_{sc}, by substituting

$$\frac{\sigma EP}{kA} T_{sr}^4 + \frac{hP}{kA} T_{sc} = \frac{\sigma EP}{kA} T_s^4 + \frac{hP}{kA} T_s \tag{51}$$

The same procedure can also be used when the heat generation term depends on temperature in some specified from, namely if it varies linearly or to the fourth power of temperature. To illustrate this the case when the heat generation is linearly dependent on temperature will be considered.

Heat Generation That is Linearly Dependent on Temperature

If the heat generation term is given by $q''' = q_o''' + \alpha T$ a temperature T_m will be defined by:

$$q_o''' = \frac{\sigma EP}{A} (T_m^4 - T_s^4) + \frac{hP}{A} (T_m - T_s) - \alpha T_m \tag{52}$$

Substituting for q''' in equation (37) gives:

$$\frac{d^2 T}{dx^2} - \frac{\sigma EP}{kA} (T^4 - T_m^4) - \left(\frac{hP}{kA} - \alpha\right) (T - T_m) = 0 \tag{53}$$

which is in the same form as equation (2).

GENERAL SOLUTION OF THE CONSTANT AREA FIN WITH TEMPERATURE DEPENDENT PROPERTIES AND HEAT GENERATION

In the following, the constant area fin will be considered in general assuming that the physical properties and the heat generation are only functions of temperature. Under these conditions, equation (1) can be written as:

$$\frac{d}{dx}\left[k(T) \frac{dT}{dx}\right] - \frac{\sigma E(T)P}{A} (T^4 - T_s^4) - \frac{h(T)P}{A} (T - T_s) + q'''(T) = 0 \tag{54}$$

From the previous consideration of the constant properties problem, it is clear that it is desirable to choose the origin at the point where the temperature gradient is known instead of where the temperature is known. Assuming the temperature to be T_o at $x = 0$ and introducing the variable $\theta = T/T_o$, equation (53) can be written as:

$$\frac{d}{dx}\left[k(T_o, \theta)\frac{d\theta}{dx}\right] = \frac{\sigma E(T_o, \theta) P T_o^3}{A}(\theta^4 - \theta_s^4) + \frac{h(T_o, \theta)P}{A}(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o} \tag{55}$$

Multiplying equation (55) by $k(T_o, \theta)$ and integrating, the following equation is obtained:

$$\left[k(T_o, \theta)\frac{d\theta}{dx}\right]^2 - \left[k(T_o, 1)\left(\frac{d\theta}{dx}\right)_{\theta = 1}\right]^2$$
$$= 2\int_1^\theta k(T_o, \theta)\left[\frac{\sigma P T_o^3}{A}E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A}h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right]d\theta \tag{56}$$

This gives

$$k(T_o, \theta)\frac{d\theta}{dx} = \pm\left\{\left[k(T_o, 1)\left(\frac{d\theta}{dx}\right)_{\theta = 1}\right]^2\right.$$
$$\left. + 2\int_1^\theta k(T_o, \theta)\left[\frac{\sigma P T_o^3}{A}E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A}h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right]d\theta\right\}^{\frac{1}{2}} \tag{57}$$

The positive sign applies when θ increses in the direction of increasing x, and the negative sign applies when θ decreases in the direction of increasing x. The solution to equation (57) can be written as:

$$x = \pm\int_1^\theta \frac{k(T_o, \theta)\,d\theta}{\left\{\left[k(T_o, 1)\left(\frac{d\theta}{dx}\right)_{\theta = 1}\right]^2 + 2\int_1^\theta k(T_o, \theta)\left[\frac{\sigma P T_o^3}{A}E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A}h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right]d\theta\right\}^{\frac{1}{2}}} \tag{58}$$

The solution to the above equation is subject to these boundary conditions, $(dT/dx)_{T = T_o}$ is constant or function of T_o and $T = T_L$ at $x = L$. Assuming a value of T_o, $(d\theta/dx)_{\theta = 1}$ can be evaluated, and in order to satisfy the second boundary condition,

$$L = \pm \int_1^{\theta_L} \frac{k(T_o, \theta)\, d\theta}{\left\{\left[k(T_o, 1)\left(\frac{d\theta}{dx}\right)_{\theta = 1}\right]^2 + 2\int_1^{\theta} k(T_o, \theta)\left[\frac{\sigma P T_o^3}{A} E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A} h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right] d\theta\right\}^{\frac{1}{2}}} \tag{59}$$

Once the value of T_o is determined that satisfies equation (59), equation (58) can be used to determine the temperature distribution. The integral on the right hand side of both equations (59) and (58) can be very readily evaluated numerically or graphically for $(d\theta/dx)_{\theta = 1} > 0$ with the other variables specified. For $(d\theta/dx)_{\theta = 1} = 0$ a singularity exists at $\theta = 1$. However, the singularity can be removed through integration by parts giving:

$$\pm \frac{x}{\sqrt{2}} = \frac{\left\{\int_1^{\theta} k(T_o, \theta)\left[\frac{\sigma P T_o^3}{A} E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A} h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right] d\theta\right\}^{\frac{1}{2}}}{\left[\frac{\sigma P T_o^3}{A} E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A} h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right]} \qquad (60) \tag{60}$$

$$+ \int_1^{\theta} \frac{\left\{\int_1^{\theta} k(T_o, \theta)\left[\frac{\sigma P T_o^3}{A} E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A} h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right] d\theta\right\}^{\frac{1}{2}} \left\{\frac{d}{d\theta}\left[\frac{\sigma P T_o^3}{A} E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A} h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right] d\theta\right\}}{\left[\frac{\sigma P T_o^3}{A} E(T_o, \theta)(\theta^4 - \theta_s^4) + \frac{P}{A} h(T_o, \theta)(\theta - \theta_s) - \frac{q'''(T_o, \theta)}{T_o}\right]^2}$$

GENERALIZATION OF THE SOLUTION TO THE PROBLEM

From the previous consideration of the special cases, the general method for handling this problem has become quite clear. It is generally advantageous, except for some cases of infinite length, to locate the zero x axis at the point where the temperature gradient is specified and not where the temperature is specified. Assuming the temperature at $x = 0$ to be T_o and substituting for $\theta = T/T_o$ in equation (1) gives:

$$\frac{d}{dx}\left[A(x)\, k(T_o, \theta)\frac{d\theta}{dx}\right] + \frac{q'''(T_o, \theta, x)\, A(x)}{T_o} \tag{61}$$

$$- P(x)\frac{dS}{dx}\left[\sigma T_o^3 E(T_o, \theta)(\theta^4 - \theta_s^4) + h(T_o, \theta, x)(\theta - \theta_s)\right] = 0$$

Multiplying by $A(x)\, k(T_o, \theta)$ gives:

$$A(x)\, k(T_o, \theta) \frac{d}{dx}\left[A(x)\, k(T_o, \theta) \frac{d\theta}{dx}\right] \tag{62}$$

$$= P(x)\, A(x) \frac{dS}{dx} k(T_o, \theta)\left[\sigma T_o^3 E(T_o, \theta)(\theta^4 - \theta_s^4) + h(T_o, \theta, x)(\theta - \theta_s)\right] - \frac{A^2(x)\, k(T_o, \theta)\, q'''(T_o, \theta, x)}{T_o}$$

Equation (62) can be integrated once giving

$$\frac{1}{2}\left[A^2(x)\, k^2(T_o, \theta)\left(\frac{d\theta}{dx}\right)^2 - A^2(x = 0)\, k^2(T_o, 1)\left(\frac{d\theta}{dx}\right)^2_{\theta = 1}\right] \tag{63}$$

$$= \int_1^{\theta} P(x)\, A(x) \frac{dS}{dx} k(T_o, \theta)\left[\sigma T_o^3 E(T_o, \theta)(\theta^4 - \theta_s^4) + h(T_o, \theta, x)(\theta - \theta_s)\right] d\theta - \int_1^{\theta} \frac{A^2(x)\, k(T_o, \theta)\, q'''(T_o, \theta, x)}{T_o}\, d\theta$$

If the right-hand side of equation (63) is only a function of θ, the integral in the right-hand side of equation (63) can be evaluated exactly, although the evaluation may have to be done numerically or graphically. A second integration would complete the solution. A value T_o is to be found to satisfy the boundary condition $\theta = \theta_L$ at $x = L$. The details of this method were explained in dealing with the constant area case with temperature dependent properties.

If the right-hand side of equation (63) is both θ and x dependent, the following scheme is used for the solution: A value of T_o is assumed that allows the calculation of θ_L. From the boundary condition at $x = 0$, $(d\theta/dx)_{\theta = 1}$ can be evaluated. Using a suitable θ increment and a suitable numerical integration scheme, equation (62) can be solved numerically, resulting in the evaluation of $d\theta/dx$. Successively integrating the result numerically gives x as a function of θ. If at $\theta = \theta_L$, $x = L$, the assumed value of T_o is the correct value, and the solution is completed. If at $\theta = \theta_L$, $x \neq L$, a new value of T_o is assumed until the solution is completed. When $x < L$ at $\theta = \theta_L$, the new assumed value of T_o must be less than the originally assumed value. When $x > L$ at $\theta = \theta_L$, the new assumed value of T_o must be greater than the originally assumed value. The proper sign for $d\theta/dx$ should be chosen depending on whether θ increases or decreases in the increasing direction of x.

CONCLUSIONS

The nonlinear differential equation that describes one-dimensional, steady-state heat exchange by both convection and radiation between a rod or a fin and the surroundings was examined in general. It was shown that the constant area case, which is of practical importance, can be solved exactly for either constant or temperature dependent physical properties and heat generation.

The conditions under which the problem in general could be solved exactly and the method of solution were considered, and a general method for solving the problem numerically was outlined.

The results obtained can be used for solving many problems of practical engineering importance as well as for design optimization. The solution also makes it possible to determine the physical properties of materials and examine the validity of some of the theories about radiant energy emission.

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APPENDIX[1]

COMPUTER PROGRAM

Two separate computer programs were written to evaluate the functions Φ and Λ. The programs are written in Fortran IV for the IBM 7094 computer.

The integrals are evaluated numerically by the Runge-Kutta method of integration. An integration interval of .001 is used until the independent variable of integration becomes equal to or greater than .98, then an integration step of .0001 is used.

The Computer Program for Evaluating the Function Φ and λ_L for the Minimum Mass Fin Geometry

For the case where g = 0, the computer program evaluates equation (24). For all other cases the program evaluates equation (23).

When $\theta_s = 1$ and $g = 0$, equation (24) has a singularity at $\lambda = 1$. For this case the upper limit of integration is set equal to .999.

When the integration is completed, the program computes λ_L for each of the equations (44) and (46) which are the minimum mass expressions for rectangular and cylindrical geometry, respectively. This is accomplished by evaluating the right hand side of equations (45) and (47) for $0 \leqq \lambda \leqq \lambda_L$, using the same increment for $\lambda^{3/2}$ that was used in the integration process, and comparing the value of each function to the difference $\Phi(1, \theta_s, r, C) - \Phi(\lambda, \theta_s, r, C)$. When either of the values compare with this difference, $\lambda_L = \lambda$ has been found for this expression.

The difference $\Phi(1, \theta_s, r, C) - \Phi(\lambda, \theta_s, r, C)$ is plotted versus $\lambda^{3/2}$.

Explanation of Parameters and Cross-reference Between Symbols (Input and Output)

Algebraic Symbol	Fortran Symbol	Description
g	G	$5kA/2\sigma ET_o^3 \, (d\theta/dx)^2_{x=0}$
r	R, RR	$5h/2\sigma ET_o^3$
θ_s	TS	T_s/T_o
$\lambda^{3/2}$	Z	Independent variable of integration
Φ	PH	$\frac{2}{3}\int_0^{\lambda}[1 - 5\theta_s^4 \lambda^4 + r\lambda^3 (1 - 2\theta_s\lambda) + C\lambda^5]^{-1/2} d(\lambda^{3/2})$ or $\frac{8}{3}\int_0^{\lambda}[1 - 5\theta_s^4 \lambda^4 + r\lambda^3(1 - 2\theta_s\lambda) + C\lambda^5]^{1/2} (10 + r\lambda^3) d(\lambda^{3/2}) - \frac{2\lambda^{3/2} [1 - 5\theta_s^4 \lambda^4 + r\lambda^3 (1 - 2\theta_s\lambda) + C\lambda^5]^{1/2}}{5(1 - \theta_s^4 \lambda^4) + 2r\lambda^3(1 - \theta_s\lambda)}$

[1] Provided by James W. Price of the Applied Research Branch.

The Function Λ

For the case where $g = 0$, the program evaluates Λ by equation (25). For all other cases the program uses equation (20).

When $g = 0$ and $\theta_s = 1$, equation (25) has a singularity at $\theta = 1$. For this case the upper limit g integration is set equal to .999.

The value of Λ is stored after each integration step and when the upper limit is reached for the integration, the values of $\Lambda(1, \theta_s, r, C) - \Lambda(\theta, \theta_s, r, C)$ are plotted versus θ.

Explanation of Parameters and Cross-reference Between Symbols (Input and Output)

Algebraic Symbol	Fortran Symbol	Description
g	G	$5kA/2\sigma ET_o^3\ (d\theta/dx)^2_{x=0}$
r	R, R	$5h/2\sigma ET_o^3$
θ_s	TS	T_s/T_o
θ	THETA	Independent variable of integration
Λ	LAMDA	$\int_1^\theta [\theta^5 - 5\theta_s^4\theta + r(\theta^2 - 2\theta_s\theta) + C]^{1/2}\, d\theta$

or

$$\frac{2[\theta^5 - 5\theta_s^4\theta + r(\theta^2 - 2\theta_s\theta) + C]^{1/2}}{5(\theta^4 - \theta_s^4) + 2r(\theta - \theta_s)} + \frac{2C^{1/2}}{5\theta_s^4 + 2r\theta_s}$$

$$4\int_0^\theta \frac{[\theta^5 - 5\theta_s^4\theta + r(\theta^2 - 2\theta_s\theta) + C]^{1/2}\,(10\theta^3 + r)\,d\theta}{[5(\theta^4 - \theta_s^4) + 2r(\theta - \theta_s)]^2}$$

Data sheets are the same as previously shown.

COLUMN	ITEM	DESCRIPTION
Data Sheet Card No. 1		
1 - 45	Title	Any desired information
Data Sheet Card No. 2		
1 - 3	IP	Print interval
4 - 6	IR	Number of values of R maximum value is 18. Minimum value is 1.
Data Sheet Card No. 2		
1 - 12	G	
13 - 24	TS	
25 - 36	R1	First value in R array
37 - 48	R2	Second value in R array
49 - 60	R3	Third value in R array
61 - 72	R4	Fourth value in R array
Data Sheet Card No. 3		
1 - 12	R5	etc.

COMPUTER LISTING FOR EVALUATING Λ

```
CLAMDA(Z,TS,R,C)
      CØMMØN TS,TSM,YP1,C,R,G
      CØMMØN RR(18)
      CØMMØN TT(510),YY(6,510),DPH(510)
      DIMENSIØN TITLE(12)
      DIMENSIØN LJ(6)
      DIMENSIØN BCDX(12),BCD1(12)
      DATA BCD1/72HZ
     1                          /
      DATA BCDX/72H(LAMDA(1)-LAMDA(Z))**2
     1                          /
       READ(5,1) TITLE
  897 READ(5,500)  IP,IR
      IF(IP) 975,975,900
  900 READ(5,800) G,TS,(RR(I),I = 1,IR)
      IRR = 0
      KK = 0
      TT(1) = 0.
      IF(TS - 1.) 205,204,205
  204 TL = .999
      GØ TØ 206
  205 TL = 1.
  206 TSM = TS**4.
      DTT = .0001
      NEQ = 1
       KEY = 0
      KJ = 0
  950 KK = KK + 1
      KL = KK - 1
       IRR = IRR + 1
      KEY = KEY + 1
      R = RR(IRR)
      DT =.001
       C= G + 5.*TSM-1.+R*(2.*TS - 1.)
      IF(G) 200,201,200
  200 CØN1 = 0.
       GØ TØ 202
  201 CØN1 = (2.*SQRT (C))/(5.*TSM + 2.*R*TS)
  202 THETA = 0.
      TIN = .98
      XNUMB = 1.
      Y = 0.
      TF = .001
       J = 1
      YY(KK,J) =0.
      IPP = 0
   10 CALL DIFFE(THETA,Y,YP,DT,NEQ)
       IF(THETA - TF) 10,105,105
  105 IF(THETA -.979 )110,110,108
  108 DT = DTT
      THETA = TF
      XNUMB = 10.
  110 TF = THETA + XNUMB*DT
      IPP = IPP + 1
      IF(IPP - IP) 20,25,25
   25 IPP = 0
      J = J + 1
```

```
      YY(KK,J) = 4.*Y+CON1 + 2.*YP1
      IF(KKY - 1) 30,30,35
   35 IF(LJ(KL) - J) 30,20,20
   30 TT(J) = THETA
   20 IF(TP - TL) 10,10,1000
 1000 CONTINUE
      LJ(KK) = J
      IF(IPP) 880,885,880
  880 J = J + 1
      YY(KK,J) = 4.*Y+ CON1 + 2.*YP1
      IF(KKY - 1) 40,40,45
   45 IF(LJ(KL) - J) 40,885 ,885
   40 TT(J) = THETA
      LJ(KK) = J
  885 DO 1225 K = 1,J
      DPH(K) = (YY(KK,J) - YY(KK,K))**2.
 1225 CONTINUE
      NP = LJ(KK)
      CALL QUIK3V(-1,42,LOOX,SCD1,-NP,DPH(1),TT(1))
      IF(KK - 6) 898,899,899
  898 IF(IRR - IR) 950,899,899
  899 DO 50 I = 1,KK
      IF(J - LJ(I)) 55,50,50
   55 J = LJ(I)
   50 CONTINUE
      IPNT = J/51 + 1
      DO 1200 JJ = 1,IPNT
      IF(JJ - IPNT)1205,1201,1201
 1201 LPP = J -(IPNT-1)*51
      GO TO 1210
 1205 LPP = 51
 1210 MJ = KJ*6 + 1
      LL = (JJ - 1) *51 + 1
      LP = LL+ LPP - 1
      WRITE(6,2) TITLE,TS,G, (RR(NJ),NJ=MJ,IRR)
      WRITE(6,1450)
      DO 850 L =LL,LP
      WRITE(6,3) TT(L), (YY(NN,L),NN =1,KK)
  850 CONTINUE
 1200 CONTINUE
      IF(TL - 1.) 1400,1500,1500
 1400 FT = 1.
      WRITE(6,5) FT
 1500 KK=0
      KJ = KJ + 1
      IF(IRR - IR) 950,897,897
  975 STOP
    3 FORMAT(15X,F10.4,6F14.6)
  500 FORMAT(2I3)
    1 FORMAT(12A6)
 1450 FORMAT(20X,1HZ)
  800 FORMAT(6E12.0)
    2 FORMAT(1H1,53X,12A6//18X,4HTS =,F14.4,2X,3HG =,F14.4//19X,3H   R,
     11X,(6F14.4))
      END
```

```
$IBFTC YFU
      SUBRØUTINE YFUNC(THETA,Y,YP,I)
      CØMMØN TS,TSM,YP1,C,R,G
      TCUB = THETA**3.
      YPØ = 5.*(THETA**4. - TSM) + 2.*R*(THETA - TS)
      YP2 = THETA*(-5.*TSM- 2.*TS*R + THETA*(R + TCUB)) + C
      IF(YP2) 10,20,20
   10 IF(THETA - 1.) 20,15,15
   15 YP2 = 0.
   20 IF(G) 21,25,21
   21 YP = 1./ SQRT (YP2)
      YP1 = 0.
       GØ TØ 30
   25 YP1 = SQRT (YP2) /YPØ
      YP = (YP1*(10.*TCUB + R))/YPØ
   30 RETURN
      END
```

```
$IBFTC DIF
       SUBRØUTINE DIFFE(X,Y,YP,DX,NEQ)
C RUNGE KUTTA FØURTH ØRDER SUB
C  PRICE
       DIMENSIØN Y(3),YP(3),YØ(3),XK(3,4)
       XØ = X
       DØ 1 J=1,NEQ
     1 YØ(J) = Y(J)
       DØ 10 I=1,4
       GØ TØ (7,2,3,5),I
     2 X = X + .5*DX
     3 DØ 4 J=1,NEQ
     4 Y(J) = YØ(J) + .5*XK(J,I-1)
       GØ TØ 7
     5 X = X + .5*DX
       DØ 6 J=1,NEQ
     6 Y(J) = YØ(J)+XK(J,3)
     7 CALL YFUNC(X,Y,YP,I)
       DØ 8 J=1,NEQ
     8 XK(J,I) = YP(J)*DX
    10 CØNTINUE
       DØ 11 J=1,NEQ
    11 Y(J) = YØ(J) + (XK(J,1) +2.*XK(J,2)+2.*XK(J,3)+XK(J,4))/6.
       X = XØ + DX
       RETURN
       END
```

COMPUTER LISTING FOR EVALUATING Φ AND λ_L FOR THE MINIMUM MASS FIN

```
C PH(Z,TS,R,C)
      DIMENSION LJ(6)
      COMMON TS,TS4,CON,C,R,G
      COMMON ZMIN1(6),ZMIN2(6),DPH(510)
      COMMON RR(18),ZI(510),PH(6,510)
      DIMENSION BCDX(12),BCD1(12)
      DATA BCDX/72H(PH(1)-PH(Z))**2
     1                             /
      DATA BCD1/72HZ
     1                             /
      EX2 = 1.5
      EX1 = .66666667
      DIMENSION TITLE(12)
      READ(5,400) TITLE
   10 READ(5,2) IP,IR
      IF(IP) 975,975,974
  974 READ(5,1)   G,TS,(RR(I),I =1,IR)
C     LET Z = XLAMDA**(3/2)
      IF(G) 800,801,800
  800 CZ = .66666667
      GO TO 802
  801 CZ = 2.6666667
  802 TS4 = TS**4
      KK = 0
      ZI(1) = 0.
      KJ = 0
      KEY = 0
      IF(TS - 1.) 220,215,220
  215 TL = .999
      GO TO 225
  220 TL = 1.
  225 IRR = 0
  301 KK = KK + 1
      IRR = IRR + 1
      KL = KK - 1
      R = RR(IRR)
      KEY = KEY + 1
      C=G+5.0*TS4-1.+R*(2.0*TS-1.0)
      DZZZ=.0001
      J = 1
      DZ=.001
      PHI=0.
      DPHI=0.
      Z=0.
      NEQ=1
      DZZ=.001
      ZF=DZ
      PH(KK,J) = 0.
      IPP = 0
      XNUB = 1.
  950 CALL DIFFE(Z,PHI,DPHI,DZ,NEQ)
  979 IF(Z - ZF) 950,990,990
  990 IF(Z-.979) 993,991,991
  991 DZ=DZZZ
      Z = ZF
      XNU  = 10.
      GO TO 994
```

```
993 DZ=DZZ
994 ZF = Z + XNUB*DZ
    IPP = IPP + 1
    IF(IPP - IP) 300,240,240
240 IPP = 0
    J = J+1
    PH(KK,J) = C2*PHI - 2.0*Z*CON
    IF(KEY - 1) 250,250,245
245 IF(LJ(KL) - J) 250,300,300
250 ZT(J) = Z
300 IF(ZF - ZL) 950,950,999
999 CONTINUE
    LJ(KK) = J
    IF(IPP) 255,265,255
255 J = J + 1
    PH(KK,J) =  C2*PHI - 2.*Z*CON
    IF(KEY - 1) 260,260,256
256 IF(LJ(KL) - J) 260,265,265
260 ZT(J) = Z
    LJ(KK) = J
265 IT1 = 0
    IT2 = 0
    FRCL = 0
    FRDL = 0
860 DO 890 L= 1,J
    DI = PH(KK,J) - PH(KK,L)
    Z2 = ZT(L)*ZT(L)
    Z23 = ZT(L)**EX1
    FN1 = ZT(L)*SQRT (1. + Z2*(R-Z23*(2.*TS*R + 5.*TS4 - C*Z23)))
    IF(IT1) 900,900,906
900 FRD1 = 1. + Z2*(4.*R - Z23*(10.*R*TS + 25.*TS4- 6.*C*Z23))
    FRD = - EX1*FN1/FRD1
    IF(FRD) 906,906,700
700 IF(FRD - DI) 905,905,906
905 DEL1 = DIL- FRDL
    DEL2 = FRD - DI
    ZMIN1(KK) = ZT(L) - (DEL2*(ZT(L) - ZT(L-1))/(DEL1 + DEL2))
    IT1 = 1
906 IF(IT2) 908,908,912
908 FRC1 = EX2 + Z2*(4.0*R - Z23*(R*TS*9.6666667 + 24.166667*TS4
   1 - 5.6666667*C*Z23))
    FRC = -FN1/FRC1
    IF(FRC) 912,912,705
705 IF(FRC - DI) 909,909,912
909 IT2 = 1
    DEL1 = DIL - FRCL
    DEL2 = FRC - DI
    ZMIN2(KK) = ZT(L) - (DEL2*(ZT(L) - ZT(L-1))/(DEL1+DEL2))
912 DIL = DI
    FRDL =FRD
    FRCL = FRC
    IF(IT1 - 1) 890, 270,890
270 IF(IT2 - 1) 890,1002,890
890 CONTINUE
1002 DO 1225 K = 1,J
    DPH(K) = (PH(KK,J) - PH(KK,K))**2.
1225 CONTINUE
    NP = LJ(KK)
    CALL OUTR3V(-1,42,BCDX,BCD1,-NP,DPH(1),ZT(1))
    IF(KK - 6) 275,280,280
275 IF(IRR - IR)301,280,280
```

```
  280 DØ 285 I = 1,KK
      IF(J - LJ(I)) 281,285,285
  281 J = LJ(I)
  285 CØNTINUE
      IPNT = J/51 + 1
      DØ 1200 JJ = 1,IPNT
      IF(JJ - IPNT)1205,1201,1201
 1201 LPP = J -(IPNT-1)*51
      GØ TØ 1210
 1205 LPP = 51
 1210 MJ = KJ*6 + 1
      LL = (JJ - 1) *51 + 1
      LP = LL+ LPP - 1
      WRITE(6,405) TITLE,TS,G,(RR(NJ),NJ=MJ,IRR)
      WRITE(6,1450)
      DØ 850 L =LL,LP
      WRITE(6,410) ZT(L), (PH(NN,L),NN=1,KK)
  850 CØNTINUE
 1200 CØNTINUE
      IF(TL - 1.) 1400,1500,1400
 1400 FT = 1.
      WRITE(6,410) FT
 1500 WRITE(6,419) TS,G
      DØ 418 I = 1,KK
      WRITE(6,420) RR(I),ZMIN1(I),ZMIN2(I)
  418 CØNTINUE
      KK = 0
      KJ = KJ + 1
       IF(IRR - IR)301,10,10
  975 STØP
 1450 FØRMAT(20X,1HZ)
  419 FØRMAT(1H1,
     1       2X,99HZMIN1 AND ZMIN2 CØRRESPØND TØ MIN MASS FUNCTIØNS FØR
     1RECT AND CYLINDRICAL GEØMETRIES , RESPECTIVELY// 15X,4HTS =,F10.3,
     114X,3HG =,F10.3//9X,1HR,11X,5HZMIN1,9X,5HZMIN2//)
  420 FØRMAT(2X,3F14.6)
  410 FØRMAT(13X,F10.4,6F14.6)
  405 FØRMAT(1H1,53X,12A6//18X,4HTS =,F14.4,2X,3HG =,F14.4//19X,3H  R,
     1 1X,(6F14.4))
    1 FØRMAT(6E12.0)
    2 FØRMAT(4I3)
  400 FØRMAT(12A6)
      END
```

```
$IBFTC YFU
      SUBROUTINE YFUNC(Z,PHI,DPHI,I)
      CØMMØN TS,TS4,CØN,C,R,G
      ZSQ=Z*Z
      ZP=Z**(2.6666667)
      ZP1=Z**(.66666667)
      DEN=5.0*(1.-TS4*ZP)+2.0*R*ZSQ*(1.-TS*ZP1)
      DNUM = 1.0 + ZSQ*(R   - ZP1*(2.0*TS*R   +5.0*TS4 - C*ZP1))
      IF(G) 9,10,9
    9 CØN = 0.
      DPHI = 1./SQRT (DNUM)
      GØ TØ 12
   10  IF(DNUM) 15,15,20
   15 DPHI = 0.
      CØN = 0.
        GØ TØ 12
   20 CØN = SQRT(DNUM)/DEN
      DPHI=CØN/DEN*(10.+R*ZSQ)
   12 RETURN
      END
```

```
$IBFTC DIF
      SUBROUTINE DIFFE(X,Y,YP,DX,NEQ)
C RUNGE KUTTA FOURTH ØRDER SUB
C  PRICE
      DIMENSIØN Y(3),YP(3),YØ(3),XK(3,4)
      XØ = X
      DØ 1 J=1,NEQ
    1 YØ(J) = Y(J)
      DØ 10 I=1,4
      GØ TØ (7,2,3,5),I
    2 X = X + .5*DX
    3 DØ 4 J=1,NEQ
    4 Y(J) = YØ(J) + .5*XK(J,I-1)
      GØ TØ 7
    5 X = X + .5*DX
      DØ 6 J=1,NEQ
    6 Y(J) = YØ(J)+XK(J,3)
    7 CALL YFUNC(X,Y,YP,I)
      DØ 8 J=1,NEQ
    8 XK(J,I) = YP(J)*DX
   10 CØNTINUE
      DØ 11 J=1,NEQ
   11 Y(J) = YØ(J) + (XK(J,1) +2.*XK(J,2)+2.*XK(J,3)+XK(J,4))/6.
      X = XØ + DX
      RETURN
      END
```

APPROVAL

NONLINEAR HEAT TRANSFER AND TEMPERATURE DISTRIBUTION THROUGH FINS AND ELECTRIC FILAMENTS OF ARBITRARY GEOMETRY WITH TEMPERATURE-DEPENDENT PROPERTIES AND HEAT GENERATION

By A. R. Shouman

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N. E. Welch

N. E. WELCH
Chief, Research Analysis Section

R. R. Head

R. R. HEAD
Chief, Applied Research Branch

H. G. Paul

H. G. PAUL
Chief, Propulsion Division

W. R. Lucas

W. R. LUCAS
Director, Propulsion and Vehicle Engineering Laboratory

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